

Eigenvalue Analysis of Mindlin Plates Resting on Elastic Foundation

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ABSTRACT The purpose of this paper is to study free vibration analysis of thick plates resting on Winkler foundation using Mindlin's theory with shear locking free fourth order finite element, to determine the effects of the thickness/span ratio, the aspect ratio, subgrade reaction modulus and the boundary conditions on the frequency parameters of thick plates subjected to free vibration. In the analysis, finite element method is used for spatial integration. Finite element formulation of the equations of the thick plate theory is derived by using higher order displacement shape functions. A computer program using finite element method is coded in C++ to analyze the plates as free, clamped or simply supported along all four edges. In the analysis, 17-noded finite element is used. Graphs are presented that should help engineers designing of thick plates subjected to earthquake excitations. It is concluded that 17-noded finite element can effectively be used in the free vibration analysis of thick plates. It is also concluded that, the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio.

Keywords: Free Vibration Analysis, Winkler Foundation, Thick Mindlin Plates, Shear Locking Free Element, Finite Element Method

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1. INTRODUCTION

The plates resting on elastic foundation is one of the most popular topics for the last decade in many engineering application. Winkler model, Pasternak model, Hetenyi model, Vlasov and Leont'ev model are the models used by the researchers to calculate the soil effects on the plate.

The dynamic behavior of thick plates has been investigated by many researchers [1, 2, 3, 4, 5]. In many cases, numerical solution can have lack of convergence, which is known as "shear-locking". This problem can be avoided by increasing the mesh size, i.e. using finer mesh, but if the thickness/span ratio is "too small", convergence may not be achieved even if the finer mesh is used for the low order displacement shape functions.

In order to avoid shear locking problem, different methods and techniques, such as reduced and selective reduced integration, the substitute shear strain method, etc., are used by several researchers [6, 7, 8, 9, 10]. The same problem can also be prevented by using higher order displacement shape function [11]. Vibration

analysis made by [12], they presented natural frequencies and modes of rhombic Mindlin plates. Özdemir and Ayvaz [13] studied shear locking free earthquake analysis of thick and thin plates using Mindlin's theory. However, no references have been found in literature for the free vibration analysis of thick plates resting on Winkler foundation by using fourth order 17-noded finite element.

The aim of this paper is to analyze eigenvalue analysis of thick plates resting on Winkler foundation using Mindlin's theory with shear locking free fourth order finite element, to determine the effects of the thickness/span ratio, the aspect ratio, subgrade reaction modulus and the boundary conditions on the frequency parameters of thick plates with free vibration. In the study C++ computer program used for analyzing the plates which are free, clamped or simply supported along all four edges. In the code, the finite element method is used for spatial integration. Finite element formulation of the equations of the thick plate theory is derived by using fourth order displacement shape functions. In the

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analysis, 17-noded finite element is used to construct the stiffness and mass matrices since shear locking problem does not occur if this element is used in the finite element modelling of the thick and thin plates [11]. For this element in the analysis no matter what the mesh size is at the plate unless it is less than 4x4. This is a new element, details of its formulation are presented in [11] and this is the first time this element is used in the free vibration analysis of thick plates. If this element is used in an analysis, it is not necessary to use finer mesh.

2. MATHEMATICAL MODEL

The governing equation for a flexural plate (Fig. 1) subjected to free vibration without damping can be given as;

$$[M]\{\ddot{\omega}\} + [K]\{\omega\} = 0 \tag{1}$$

where [K] and [M] are the stiffness matrix and the mass matrix of the plate, respectively, ω and $\ddot{\omega}$ are the lateral displacement and the second derivative of the lateral displacement of the plate with respect to time, respectively.

The total strain energy of plate-soil-structure system (see Fig. 1) can be written as;

$$\Pi = \Pi_P + \Pi_S + V \tag{2}$$

where Π_P is the strain energy in the plate,

$$\Pi_P = \frac{1}{2} \int_A \begin{pmatrix} -\frac{\partial \varphi_x}{\partial x} & \frac{\partial \varphi_y}{\partial y} & -\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \end{pmatrix}^T E_\kappa \begin{pmatrix} -\frac{\partial \varphi_x}{\partial x} & \frac{\partial \varphi_y}{\partial y} & -\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \end{pmatrix} d_A + \tag{3}$$

$$\frac{k}{2} \int_A \begin{pmatrix} -\varphi_x + \frac{\partial w}{\partial x} & \varphi_y + \frac{\partial w}{\partial y} \end{pmatrix}^T E_\gamma \begin{pmatrix} -\varphi_x + \frac{\partial w}{\partial x} & \varphi_y + \frac{\partial w}{\partial y} \end{pmatrix} d_A$$

where Π_S is the strain energy stored in the soil,

$$\Pi_S = \frac{1}{2} \int_0^H \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_{ij} \epsilon_{ij} \tag{4}$$

and V is the potential energy of the external loading;

$$V = - \int_A \bar{q} w d_A \tag{5}$$

In this equation E_κ and E_γ are the elasticity matrix and \bar{q} shows applied distributed load.

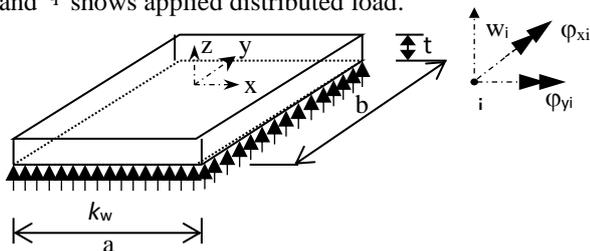


Fig. 1. The sample plate used in this study

2.1. Creating of the Stiffness Matrix

The total strain energy of the plate-soil system according to Eq. (2) is;

$$U_e = \frac{1}{2} \int_A \begin{pmatrix} -\frac{\partial \varphi_x}{\partial x} & \frac{\partial \varphi_y}{\partial y} & -\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \end{pmatrix}^T E_\kappa \begin{pmatrix} -\frac{\partial \varphi_x}{\partial x} & \frac{\partial \varphi_y}{\partial y} & -\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \end{pmatrix} d_A + \tag{6}$$

$$\frac{k}{2} \int_A \begin{pmatrix} -\varphi_x + \frac{\partial w}{\partial x} & \varphi_y + \frac{\partial w}{\partial y} \end{pmatrix}^T E_\gamma \begin{pmatrix} -\varphi_x + \frac{\partial w}{\partial x} & \varphi_y + \frac{\partial w}{\partial y} \end{pmatrix} d_A +$$

$$\frac{1}{2} \int_A (w_{x,y})^T K (w_{x,y}) d_A$$

At this equation the first and second part gives the conventional element stiffness matrix of the plate, $[k_p^e]$, differentiation of the third integral with respect to the nodal parameters yields a matrix, $[k_w^e]$, which accounts for the axial strain effect in the soil. Thus the total energy of the plate-soil system can be written as;

$$U_e = \frac{1}{2} \{w_e\}^T ([k_p^e] + [k_w^e]) \{w_e\} d_A \tag{7}$$

Where;

$$\{w_e\} = [w_1 \ \varphi_{y1} \ \varphi_{x1} \ \dots \ w_n \ \varphi_{yn} \ \varphi_{xn}]^T \tag{8}$$

Assuming that in the plate of Fig. 1 u and v are proportional to z and that w is the independent of z , one can write the plate displacement at an arbitrary x, y, z in terms of the two slopes and a displacement as follows;

$$u_i = \{w, v, u\} = \{w_0(x,y,t), z\varphi_y(x,y,t), -z\varphi_x(x,y,t)\} \tag{9}$$

where w_0 is average displacement of the plate, and φ_x and φ_y are the bending slopes in the x and y directions, respectively.

The nodal displacements for 17-noded quadrilateral serendipity element (MT17) (Fig. 2) can be written as follows;

$$w = \sum_1^{17} h_i w_i, \quad v = z\varphi_y = z \sum_1^{17} h_i \varphi_{yi}, \quad u = -z\varphi_x = -z \sum_1^{17} h_i \varphi_{xi} \tag{10}$$

$i = 1, \dots, 17$ for 17-nodedelement,

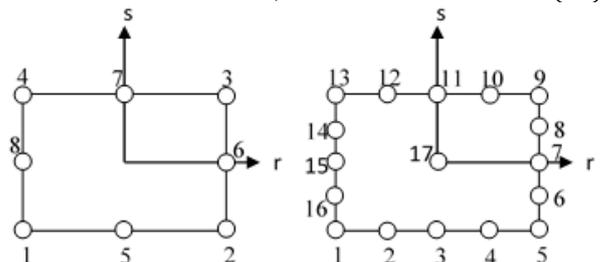


Fig. 2. 8- (second order), and 17-noded (fourth order) quadrilateral finite elements used in this study

The displacement function chosen for this element is;

$$w = c_1 + c_2r + c_3s + c_4r^2 + c_5rs + c_6s^2 + c_7r^2s + c_8rs^2 + c_9r^3 + c_{10}r^3s + c_{11}rs^3 + c_{12}s^3 + c_{13}r^2s^2 + c_{14}r^4 + c_{15}r^4s + c_{16}rs^4 + c_{17}s^4. \quad (11)$$

From this assumption, it is possible to derive the displacement shape function to be [11];

$$h = [h_1, \dots, h_{17}] \quad (12)$$

Then, the strain-displacement matrix [B] for this element can be written as follows [13]:

$$[B] = \begin{bmatrix} 0 & 0 & -\frac{\partial h_i}{\partial x} & \dots \\ 0 & \frac{\partial h_i}{\partial y} & 0 & \dots \\ 0 & \frac{\partial h_i}{\partial x} & -\frac{\partial h_i}{\partial y} & \dots \\ \frac{\partial h_i}{\partial x} & 0 & -h_i & \dots \\ \frac{\partial h_i}{\partial y} & h_i & 0 & \dots \end{bmatrix}_{5 \times 51} \quad (13)$$

$i = 1, \dots, 17$ for 17-nodedelement

The stiffness matrix for MT17 element can be obtained by the following equation [14].

$$[K] = \int_A [B]^T [D][B] dA \int_{-1}^1 \int_{-1}^1 [B]^T [D][B] J |drds \quad (14)$$

which must be evaluated numerically [10].

As seen from Eq. (14), in order to obtain the stiffness matrix, the strain-displacement matrix, [B], and the flexural rigidity matrix, [D], of the element need to be constructed.

The flexural rigidity matrix, [D], can be obtained by the following equation.

$$[D] = \begin{bmatrix} E_k & 0 \\ 0 & E_\gamma \end{bmatrix} \quad (15)$$

In this equation, [E_k] is of size 3x3 and [E_γ] is of size 2x2. [E_k], and [E_γ] can be written as follows [16, 17]:

$$E_k = \frac{t^3}{12} \begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0 \\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & \frac{E}{2(1-\nu)} \end{bmatrix};$$

$$E_\gamma = kt \begin{bmatrix} \frac{E}{2.4(1+\nu)} & 0 \\ 0 & \frac{E}{2.4(1+\nu)} \end{bmatrix} \quad (16)$$

where E, ν, and t are modulus of the elasticity, Poisson's ratio, and the thickness of the plate, respectively, k is a constant to account for the actual non-uniformity of the shearing stresses. By assembling the element stiffness matrices obtained, the system stiffness matrix is obtained.

2.2. Evaluation of the Mass Matrix

The formula for the consistent mass matrix of the plate may be written as;

$$M = \int_{\Omega} H_i^T \mu H_i d\Omega \quad (17)$$

In this equation, μ is the mass density matrix of the form [Tedesco et al., 1999]

$$\mu = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad (18)$$

where $m_1 = \rho p t$, $m_2 = m_3 = \frac{1}{12}(\rho_p t^3)$, and ρp is the mass densities of the plate, and H_i can be written as follows,

$$H_i = [dh_i / dx \quad dh_i / dy \quad h_i] \quad i = 1 \dots 17. \quad (19)$$

It should be noted that the rotation inertia terms are not taken into account. By assembling the element mass matrices obtained, the system mass matrix is obtained.

2.3. Evaluation of Frequency of Plate

The formulation of lateral displacement, w, can be given as motion is sinusoidal;

$$w = W \sin \omega t \quad (20)$$

Here ω is the circular frequency. Substitution of Eq. (20) and its second derivation into Eq. (1) gives expression as;

$$[K - \omega^2 M] \{W\} = 0 \quad (21)$$

Eq. (21) is obtained to calculate the circular frequency, ω, of the plate. Then natural frequency can be calculated with the formulation below;

$$f = \omega / 2\pi \quad (22)$$

3. NUMERICAL EXAMPLES

3.1. Data for Numerical Examples

In the light of the results given in references [17, 18], the aspect ratios, b/a , of the plate are taken to be 1, 1.5, and 2.0. The thickness/span ratios, t/a , are taken as 0.01, 0.05, 0.1, 0.2, and 0.3 for each aspect ratio. The shorter span length of the plate is kept constant to be 10 m. The mass density, Poisson's ratio, and the modulus of elasticity of the plate are taken to be $2.5 \text{ kN.s}^2/\text{m}^2$, 0.2, and $2.7 \times 10^7 \text{ kN/m}^2$. Shear factor k is taken to be $5/6$. The subgrade reaction modulus of the Winkler-type foundation is taken as 500 and 5000 kN/m^3 .

Rather than starting from the finite element network size for the sake of correctness of the results, the network size required to achieve the desired accuracy is determined before delivering any results. This analysis was performed separately for the mesh size. It has been concluded that when using 4×4 mesh size $10 \text{ m} \times 10 \text{ m}$ plate with 17-noded elements, the results have acceptable error. As in the case of the square plate, the lengths of the each element are kept constant in the x and y directions.

In order to show that the mesh density used in this paper is enough to obtain correct results, the first six frequency parameters of the thick plate with $b/a=1$ and $t/a=0.05$ is presented in Table 1 by comparing with the result obtained SAP2000 program and the results Özgan and Daloğlu [2015]. In this study Özgan and Daloğlu used 4-noded and 8-noded quadrilateral finite element with 10×10 and 5×5 mesh size. It should be noted that the results presented for MT17 element are obtained by using equally spaced 2×2 mesh size. As seen from Table 1, the results obtained by using 17-noded quadrilateral finite element have excellent agreement with the results obtained by [18] and SAP2000 software even if 2×2 mesh size is used for MT17 element.

3.2. Results

The first six frequency parameters of thick plate resting on Winkler foundation with free edges are compared with the same thick plate modeled by [18] and SAP2000 program and it is presented in Table 1. The subgrade reaction modulus of the Winkler-type foundation for this example is taken to be 5000 kN/m^3 . This thick plate is modeled with MT17 element 2×2 mesh size for $b/a=1.0$, $t/a=0.05$ ratios.

As seen from Table 1, the values of the frequency parameters of these analyses are so close even if this study mesh size is so poor. Then parameter such as aspect ratio, b/a , thickness/span ratio, t/a where taken in a wider range, and analyses were performed.

Table 1. The first five natural frequency parameters of plates for $b/a=0.1$ and $t/a=0.05$

$\lambda_i = \omega^2$	[18]	This Study	SAP2000
	PBQ8(FI)	MT17 (4 element)	
1	3990.42	4002.41	4000.00
2	3990.42	4002.41	4000.00
3	4000.40	4021.55	4000.00
4	8676.00	8650.67	8619.60
5	13957.64	13789.50	13292.31
6	17252.34	16939.10	16380.24

Table 2. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates resting on elastic foundation

		Subgrade reaction modulus $k=500$						
k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	1.0	0.05	456.73	456.73	469.98	5048.72	10235.7	13366.9
		0.10	235.42	235.42	283.12	17448.0	37556.4	49322.0
		0.20	171.76	171.76	175.32	58681.1	126694.	164490.
		0.30	149.09	149.09	179.38	109100.	229933.	295362.
	1.5	0.05	458.49	464.03	470.14	2492.94	2660.49	10937.2
		0.10	241.45	259.74	289.49	7970.41	8878.92	39117.9
		0.20	170.28	173.06	174.51	27346.7	31896.0	127052.
		0.30	153.01	163.56	183.08	52430.1	63261.4	225607.
	2.0	0.05	459.37	466.66	470.22	1161	1588.82	5784.55
		0.10	244.46	271.13	292.64	3031.39	4557.16	20388.1
		0.20	169.53	171.92	174.46	10730.8	15546.2	68560.2
		0.30	154.97	170.85	184.92	22168.0	30127.2	126587.
3.0	0.05	460.25	468.61	470.30	603.44	951.50	1519.07	
	0.10	247.47	281.97	295.67	825.36	2129.95	4437.42	
	0.20	168.79	170.78	173.44	2333.89	6956.53	15815.4	
	0.30	156.93	177.91	186.77	4859.98	13529.7	32166.7	

The first six frequency parameters of thick plates resting on Winkler foundation considered for different aspect ratio, b/a , thickness/smaller span ratio, t/a , are presented in Table 2 for with free edges and in Table 3 for the thick simply supported plates. To see the effects of these changes on the first six frequency parameters, they are also presented in Figures 3 for the thick free plates, in Figures 4 for the thick simply supported plates.

As it can be seen from Tables 2, and 3, and Figures 3, and 4, the values of the first three frequency parameters for a constant value of t/a increase as the aspect ratio, b/a , increases up to the 3rd frequency parameters, but after the 3rd frequency parameter, the values of the frequency parameters for a constant value of t/a decrease as the aspect ratio, b/a , increases.

As also seen from Tables 2, and 3, and Figures. 3, and 4, the values of the first three frequency parameters for a constant value of b/a decrease as the thickness/span ratio, t/a , increases up to the 3rd frequency parameters, but after the 3rd frequency parameters, the values of the frequency parameters for a constant value of b/a increase as the thickness/span ratio, t/a , increases.

The increase in the frequency parameters with increasing value of b/a for a constant t/a ratio reduces for larger values of b/a up to the 3rd frequency parameters. After the 3rd frequency parameters, the decrease in the frequency parameters with increasing value of b/a for a constant t/a ratio reduces for larger values of b/a .

Table 3. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick simply supported plates resting on elastic foundation

		Subgrade reaction modulus $k=500$						
		$\lambda = \omega^2$						
k	b/a	t/a	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	0.05	0.05	9047.4	53796.	53796.	132323.	208613.	208895.
		0.10	31594.4	186728.	186728.	431328.0	675755.9	678833.0
		1.0	100289.	524820.	524820.	1078904.	1617046.	1635881.
	0.20	0.30	177442.	817902.	817902.	1562458.	2233537.	2283380.
		0.05	5015.82	17094.0	43388.9	54027.36	71060.68	133204.5
		0.10	17329.0	60561.6	154092.	188245.0	242603.2	436597.4
	1.5	0.20	57457.2	186916.	449433.	530921.1	658117.1	1095658.
		0.30	106026.	318024.	715938.	828749.9	1001294.	1585462.
		0.05	3912.42	9138.54	23276.2	39998.98	54142.96	54143.89
	2.0	0.10	13407.9	32257.5	82636.3	143304.1	189004.4	189006.5
		0.20	45541.0	103507.	250665.	424221.4	533149.2	533974.8
		0.30	85976.3	183477.	417846.	682308.0	830510.8	834182.7
3.0	0.05	3220.63	5043.23	9169.12	17200.91	31325.02	37650.27	
	0.10	10941.0	17534.1	32481.8	61320.90	111087.0	135748.4	
	0.20	38015.9	58510.7	104603.	190395.9	330316.7	406397.0	
		0.30	73378.6	108073.	185541.	324384.2	539235.3	658701.9

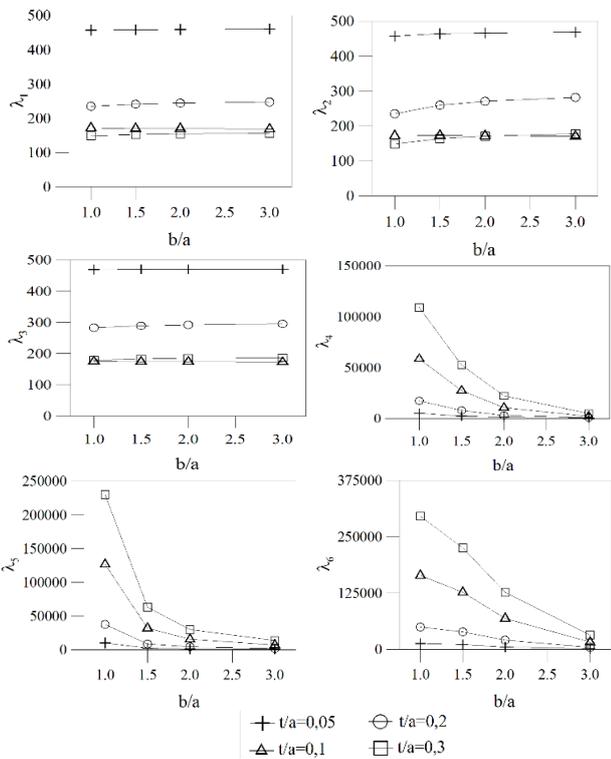


Fig. 3. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters (λ_1 to λ_6) of the thick free plates with subgrade reaction modulus $k=500$,

The changes in the frequency parameters with increasing value of b/a for a constant t/a ratio is larger for the smaller values of the b/a ratios. Also, the changes in the frequency parameters with increasing value of b/a for a constant t/a ratio is less than that in the frequency parameters with increasing t/a ratios for a value of b/a .

These observations indicate that the effects of the change in the t/a ratio on the frequency parameter of the plate are generally larger than those of the change in the b/a ratios considered in this study.

As it can also be seen from Tables 2, and 3, and Figures 3, and 4 that the curves for a constant value of b/a ratio are fairly getting closer to each other as the value of t/a increases up to the 3rd frequency parameters. This shows that the curves of the frequency parameters will almost coincide with each other when the value of the ratio of t/a increases more. After the 3rd frequency parameters, the curves for a constant value of t/a ratio are getting closer to each other as the value of b/a increases.

In other words, up to the 3rd frequency parameters, the increase in the t/a ratio will not affect the frequency parameters after a determined value of t/a . After the 3rd frequency parameters, the increase in the b/a ratio will not affect the frequency parameters after a determined value of b/a . It should be noted that the increase in the frequency parameters with increasing t/a ratios for a constant value of b/a ratio gets larger for big values of the frequency parameters.

These observations indicate that the effects of the change in the t/a ratio on the frequency parameter of the thick plates simply supported or clamped along all four edges are always larger than those of the change in the aspect ratio.

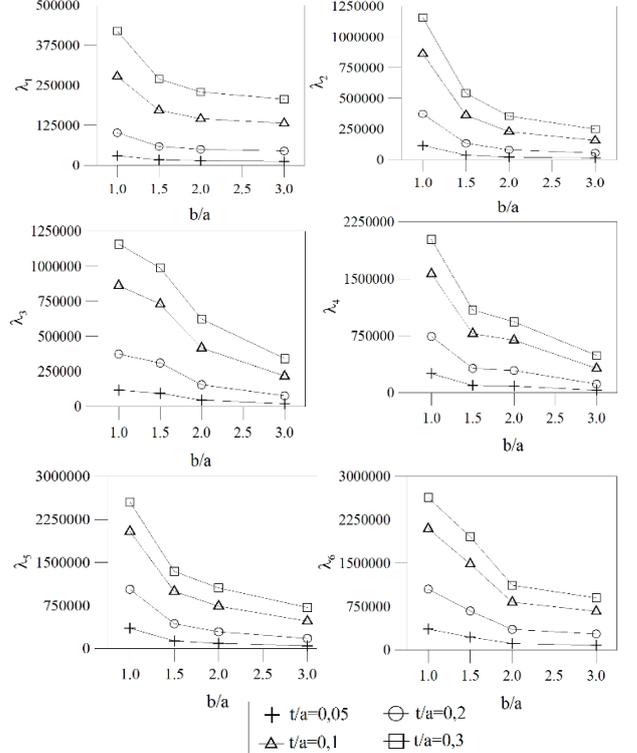


Fig. 4. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters (λ_1 to λ_6) of the thick simply supported plates with subgrade reaction modulus $k=500$,

4. CONCLUSION

The aim of this article is to study the parametric eigenvalue analysis of thick plates using Mindlin theory using high-order finite elements and to determine the effects of thickness/ span ratio, aspect ratio and boundary conditions on the linear response of applied thick plates.

As a result, free vibration analyze of the thick plates were done by using p version serendipity element, and the coded program on the purpose is effectively used. In addition, the following conclusions can also be drawn from the results obtained in this study.

The frequency parameters increases with increasing b/a ratio for a constant value of t/a up to the 3rd frequency parameters, but after that those decrease with increasing b/a ratio for a constant value of t/a .

The frequency parameters decreases with increasing t/a ratio for a constant value of b/a up to the 3rd frequency parameters, but after that those increases with increasing t/a ratio for a constant value of b/a .

The effects of the change in the t/a ratio on the frequency parameter of the thick plate are generally larger than those of the change in the b/a ratios considered in this study.

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Biography

Yaprak İtir Özdemir was born in 1975. She received her B.Sc. Degree in Civil Engineering from Karadeniz Technical University in 1997, Trabzon, Turkey. She worked in private sector a couple of years as a civil engineer. In 2001, she received her M.Sc. degree and 2007 her Ph.D. degree from Karadeniz Technical University, Trabzon, Turkey. She became an associate professor in 2012. Özdemir is currently an associate professor at Karadeniz Technical University. Her research interests include thick plates, antique engineering, and structural engineering.