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Energy eigenvalue of a one dimensional asymmetric triangle potential well

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ABSTRACT Various quantum wells have been studied as a theoretical model for the instructional of quantum physics in the early years of quantum mechanics. With the development of solid-state physics, such structures have found practical applications. Among these well models, asymmetric triangular quantum wells have started to take an important place today. Because in such wells, the asymmetry of the well can be controlled by the change in concentration in the compound. In this study, have examined the bound states and scattering states within the well by changing the parameters such as; barrier height and barrier width of the asymmetric quantum well. The scattering transfer matrix method is used as it provides convenience as a method. The results have founded differ for the asymmetric triangular quantum well compared to other symmetric quantum wells, and its practical applications are discussed.

Keywords: Quantum wells, Transfer matrix method, Heterostructures

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1. INTRODUCTION

Low dimensional systems are of fundamental importance in quantum physics, materials science and quantum chemistry, quantum biology, and optoelectronics. In contrast to more conventional three-dimensional materials, in low-dimensional systems, the quantum effects in low dimensions are predominant, leading to exotic quantum phenomena. Quantum effects will be of fundamental importance in biology in the coming years [1]. Apart from these, even the theoretical examination of low dimensional systems is instructive in understanding quantum physics. Low-dimensional systems can be classified as quantum well, quantum wire, and quantum dot. Quantum wells are gaining importance in the fields of application of quantum physics [2-5]. Quantum well problems have found application in many areas such as; solid-state, atom, and nuclear physics. It is important especially for the study of heterostructures [6-10]. In practice, wells and barriers are formed using heterostructures such as GaAs / GaAs. With recent advances in nanoscale device manufacturing technology, high-quality resonance tunneling structures in different geometries (rectangular, parabolic, triangle, etc.) can be produced by various researchers. [11].

The triangular potential well is one of the most widely used geometries because the profile of the potential function in semiconductor quantum structures such as the well-known modulation-doped AlGaAs-GaAs heterojunction is almost triangular for electrons in GaAs. Among all the heterostructures, this potential well is probably the most researched structure. On the other hand, another very important situation in which an almost triangular shaped well is formed occurs in a semiconductor in a MOS structure. Semiconductor distributed feedback (DFB) lasers in the GaAs-AlGaAs material system are attractive sources for many applications, including pump sources for the optical communication system and quantum sensors. The DFB laser is a surface ridge waveguide laser with the schematic diagram shown in Figure 1 [19].

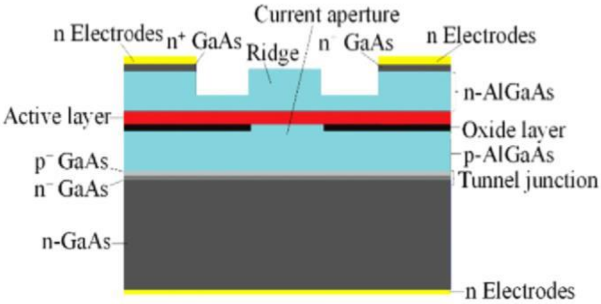

Fig. 1. Schematic diagrams of the DFB laser structure cross-section [19]

Al_xGa_{1-x}/GaN heterostructures are very important for high power, high frequency, high-temperature field interactive transistor fabrication. Triangular potential wells are formed in single heterostructures, and the Schrödinger equation in the vibrational method is one of the approximate solutions to solve it [12]. Pödör used an exponential function as the trial wave function in the variational method and a parameter ***b*** as the variable. 0.1% difference between the energy values calculated with this approach, and the actual value was reported. Verma et al. [13] numerically solved the double triangular potential well problem. They reported that the symmetrical triangular well was formed in the InAlAs / GaAs double gate. In the study by Altuğ [14], the cross-over coefficient, and the energy spectrum were investigated in one-dimensional quantum wells and barriers. Svetlana et al. [15] studied the energy eigenvalues in the infinite triangle well in the electric field.

There are studies in the literature that electron transport in structures with different geometries was investigated. One of these methods is the transfer matrix method. The electronic transport in the inverse parabolic double barrier structure was investigated by Batı et al. [16]. Esaki transfer matrix method was used to investigate the resonance tunneling [17]. Guang used the variational method for the ground state of donor and light and heavy hole excitations in triangular quantum wells. [18]. Nevertheless, the literature on the asymmetric triangular quantum well is quite limited. Chen et al. studied the double asymmetric quantum well [19]. Qiang et al. studied the subband transitions from one (001) GaAs / GaAlAs asymmetric triangular quantum well (ATQW) structure at 300 and 80 Kelvin using contactless electro modulation photo reflectance and non-contact electro reflectance methods [20].

In this study, the double tunnel barrier, the permeability coefficient as a function of energy, was calculated for the GaAs / AlGaAs using the transfer matrix method. In such studies, the transfer matrix method is generally used for the transport of an electron in quantum wells. In this study, electron transfer and energy levels in one-dimensional finite asymmetric triangular quantum wells are investigated using the transfer matrix method. The purpose of this study is to create asymmetric triangular wells by changing the x value on one side of the well in GaAs / AlxGa1-xAs. In practice, it is important to know how to change energy eigenvalues in such a well.

2. THEORETICAL CALCULATIONS

Schematic representation of asymmetric triangular potential well is shown in Figure 2. The Schrödinger equation can be written for each region in Figure 2 as Equations 1 to 4.

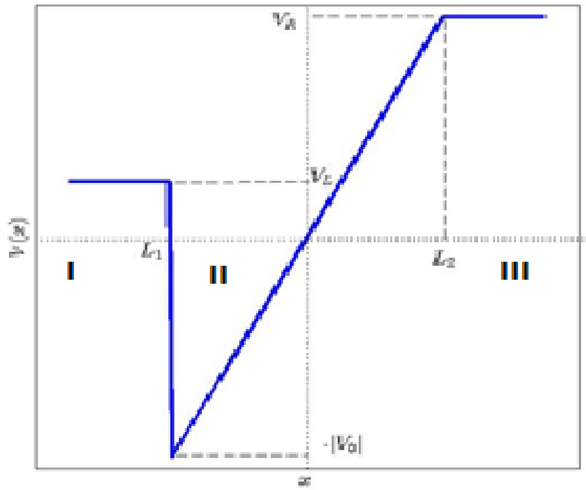

Fig. 2. Schematic representation of asymmetric triangular potential well

If E > V_R case is solved according to the Schrödinger wave equation;

1. Region: $\psi_1 = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x}$ (1)

3. Region: $\psi_3 = A_3 e^{ik_3 x} + B_3 e^{-ik_3 x}$ (2)

2. Solution for region 2:

$$V(x) + |V_0| = \frac{|V_R+V_0|}{|L_2-L_1|}(X - L_1) \quad (3)$$

$$\frac{1}{2}\frac{\partial^2\psi}{dx^2} + \left[\frac{(V_R+V_0)}{(L_2-L_1)}(X-L_1) - V_0\right]\psi = E\psi \quad (4)$$

If all the operators of Equation (3) and (4) are multiplied by two, the updated version of the equation can be given as follows;

$$\frac{d^2\psi}{dx^2} = \underbrace{\left[2E - \frac{2(V_R+V_0)}{(L_2-L_1)}(X-L_1) - 2V_0\right]}\psi = 0 \quad (5)$$

$$z = 2E + 2V_0 - \frac{2(V_R+V_0)}{(L_2-L_1)}(X-L_1) \quad (6)$$

$$\frac{dz}{dx} = \underbrace{-\frac{2(V_R+V_0)}{(L_2-L_1)}}_{\alpha}$$

If we call $\alpha = -\frac{2(V_R+V_0)}{(L_2-L_1)}$. The expression is;

$$\frac{d^2\psi}{dz^2} - \frac{1}{\alpha^2} z\psi = 0 \quad (7)$$

$$\psi_3 = A_2 A_{\text{i}}(\alpha z) + B_2 B_i(\alpha z) \quad (8)$$

If we use the boundary conditions;

$$M_1 = \frac{1}{2}\begin{bmatrix} e^{-ik_L L_1}\left(Ai + \frac{\alpha Z_1{}'}{ik_L}Ai'\right) & e^{-ik_L L_1}\left(Bi + \frac{\alpha Z_1{}'}{ik_L}Bi'\right) \\ e^{ik_L L_1}\left(Ai - \frac{\alpha Z_1{}'}{ik_L}Ai'\right) & e^{ik_L L_1}\left(Bi - \frac{\alpha Z_1{}'}{ik_L}Bi'\right) \end{bmatrix}$$

(9)

$$M_2 = \pi \begin{bmatrix} e^{ik_R L_2}\left(Bi' + \frac{ik_R}{\alpha Z_2} Bi\right) & e^{-ik_R L_2}\left(Bi' + \frac{ik_R}{\alpha Z_2} Bi\right) \\ e^{ik_R L_2}\left(Ai' - \frac{ik_R}{\alpha Z_2} Ai\right) & e^{-ik_R L_2}\left(Ai' + \frac{ik_R}{\alpha Z_2} Ai\right) \end{bmatrix} \quad (10)$$

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = M_1 \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} \quad (11)$$

and transfer matrix (TM) is;

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = M_2 \begin{pmatrix} A_3 \\ B_3 \end{pmatrix}, \qquad \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = TM \begin{pmatrix} A_3 \\ B_3 \end{pmatrix} \quad (12)$$

Here $TM = M_1.M_2$. The transmission probability T is calculated as Equation 13.

$$T = \frac{k_R}{k_L} \frac{1}{|TM_{11}|^2} \quad (13)$$

It was calculated based on the energy of the incoming electron by calculating the elements in the first row and first column of the transition coefficient matrix.

3. RESULTS AND DISCUSSIONS

The transition probability and the energy levels for the asymmetric triangle well depending on well width are given in Table 1. These calculations are made using the MATLAB software.

Table1. Energy levels according to well width change in a triangular quantum well. ($V_0 = 250\, meV$, $V_L = 0$, $V_R = 100\, meV$)

L_W(nm) Well width	E_1 (meV)	E_2 (meV)	E_3 (meV)
1	180	770	1630
2	100	560	1180
3	40	410	890
4	10	310	690
5	-20	240	550
6	-40	180	450
7	-60	140	370
8	-70	110	300
9	-80	80	250
10	-90	60	210
12	-110	30	150
14	-120	0	110
15	-120	-10	90
20	-140	-50	30

When Table 1 is reviewed, it is seen that the size of the bound energy levels increases as the width of the increases. When the well width becomes 5 nm, the first bound state occurs. When the well width becomes 15 nm, the second bound state occurs in the system. While the energy levels (E_1, E_2, E_3) change, the bound energy states decrease. This shows that energies are trapped inside the well as the well expands. According to Table 2, the height of the right potential barrier (V_R) increases, and the associated energy level decreases. Therefore, the energies of well level increase with increasing of the potential barrier.

Table 2. Energy levels according to the right barrier change in the triangular quantum well (V_R = $100\, meV$, $L_W = 4\, nm$, $V_L = 0$)

$V_R(meV)$	E_1 (meV)	E_2 (meV)	E_3 (meV)
-50	-50	220	620
0	-30	250	640
50	-10	280	670
100	10	310	690
150	20	340	720
200	40	370	750
250	50	400	770
300	70	430	800
400	90	480	860
500	110	540	920

It is seen from Table 3 that the energy level shifts towards the lower energy as the well depth increases. In this case, it is observed that the difference in the bound state energy levels is not linear.

Table 3. Energy levels according to the well depth variation in a triangular quantum well

$V_0(meV)$	E_1 (meV)	E_2 (meV)	E_3 (meV)
0	120	360	740
10	120	360	740
50	100	350	730
70	90	350	730
80	90	340	720
90	80	340	720
100	80	340	720
150	50	330	710
200	30	320	700
250	10	310	690
300	-20	300	690
350	-50	290	680
400	-70	280	670
450	-100	270	660
500	-120	260	650

It is observed that all high-energy electrons pass to the opposite side when the electron energies are 50, 100, and 250 meV depending on the right barrier height as seen in Figure 1. It can be concluded that the electrons corresponding to the barrier height cannot pass the energy barrier.

Figure 3 shows that there is a transition to the opposite side for electrons with an energy above the right barrier. In the energies corresponding to the well energy levels, all the high energy electrons pass through the right barrier. Moreover, the level of energy depends on the well width.

Although the height of the right energy barrier is the same as the deepening of the well, there is no change in the electron energy where the transition starts. Also, the variation in the depth of the well is noted to cause changes in the number of transferred electrons in Figure 4. The transmission of electrons dependent on well depth for the 50, 100, and 250 meV electron energies are given in Figure 5.

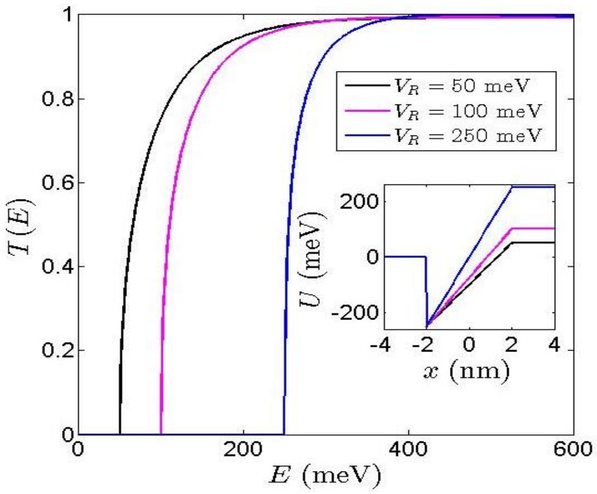

Fig. 3. The transmission of electrons due to right barrier change ($V_L = 0, L_w = 4\,nm, V_R = 50\,meV, 100\,meV, and\ 250\,meV$)

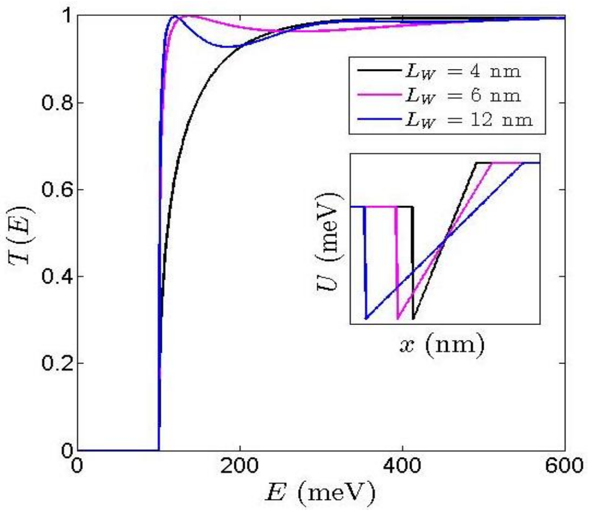

Fig. 4. The transmission of electrons dependent on well width ($V_L = 0, V_R = 100\,meV, V_0 = 250\,meV\ ve\ L_w = 4\,nm, 6\,nm, 12\,nm$)

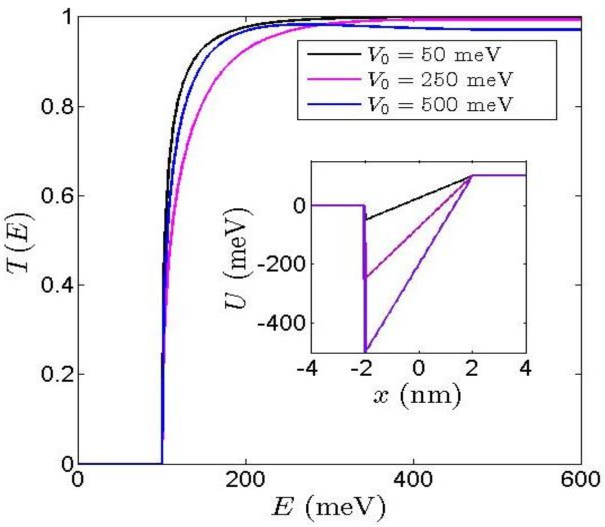

Fig. 5. The transmission of electrons dependent on well depth ($V_L = 0, V_R = 100\,meV,\ L_W = 4\,nm, V_0 = 50\,meV, 250\,meV, 500\,meV$)

4. CONCLUSION

In literature, only a small amount of asymmetric potential well problems have been studied so far. Scattering and bound conditions were not investigated in those studies. This study, different from the existing literature, aimed to investigate one dimensional finite asymmetric triangular potential well using the transfer matrix method. Parameters such as well width and electron energy in this study were based on the actual physical dimensions of the triangular wells formed in heterostructures.

The energy levels of the asymmetric triangular well potential in one dimension were determined and interpreted according to well width, asymmetry, and well depth changes. At the same time, the transmission probability is given for different structure parameters. It was noted that transmission occurs when electron energy is bigger than the potential barrier as expected. The interesting thing here is that electrons at certain electron energies pass completely through the wells in graphics. In other words, the electrons at certain voltage currents pass through as if there was no well, ie without encountering resistance. Also, there is a nonlinear relationship in the change of state energy levels. This suggests that such structures can find applications in cryptography in addition to electronic applications.

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